Physics, Mathematical Models and Human Intuition

a short essay by Andrea Pasquinucci

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In the last few centuries our understanding of the physical world in which we live has improved at incredible pace even if the approach to its description has not changed from the times of the ancient Egyptians and Greeks.

We describe physical phenomena by means of mathematical models.

It is important to know that we have developed thousands or even millions of such mathematical models. There has always been a quest for the 'Theory' which would describe and explain everything but, if it exists, it has always eluded us.

In practice each mathematical model describes a class or sub-class of physical phenomena. Let's make an example: suppose we need to describe the motion of a small rock. The first mathematical model we can think of, describes the motion of all rock's particles, one by one. Unfortunately this is a massively hard mathematical problem which we can hardly solve: the equations look simple but the amount of numbers we should crunch is typically just too large. Instead we can use another mathematical model which describes the motion of the centre of mass of the rock. This very simple mathematical model works very well until the surface of the rock hits another body. If we want to describe what happens in this case we need another mathematical model, a little more complicated, which describes the motion of the centre of mass and of the surface of the rock. If the rock splits in two when it hits another body, then we'll need to use yet another mathematical model, still a little bit more complicated. So depending on what happens to our rock, we can use different mathematical models to describe its motion and we should choose the more appropriate and easier to compute.

The mathematical models used to describe physical phenomena must satisfy various requirements which we can summarize as follows:

1. all mathematical models must specify clearly under which conditions they are valid and can be used (that is, when they give correct predictions of physical phenomena);

2. it must be possible to reproduce and verify the predictions of the mathematical models whenever we want by setting up the appropriate experiments;

3. the predictions of a mathematical model must be compatible with the ones of the other

models.

(This is in a way a brief summary of the Galileo's Scientific Method.) The last point is for now a little vague but at a minimum it means that if two models describe the same phenomena they should give the same descriptions, otherwise either one of the two models is wrong or one of the two cannot be applied to this class of phenomena being out of its domain.

Another consideration to be done is that mathematical models are usually exact: they produce exact numbers as predictions. But when we do physical experiments, our instruments are never perfectly precise so that by repeating the same experiment we always get numbers which differ even if only by very little. If an experiment is set-up really well, we most probably get a distribution of numbers which follows a Guassian law and their average is what we should compare with the prediction of the mathematical model. In a typical experiment things are usually much more complex than this. Physicists are usually quite good in analysing errors and imprecisions of experiments and in extracting mean values, error intervals and confidence levels of the results.

To sum up, we have developed a myriad of mathematical models which we use to describe and predict classes of natural phenomena in some cases with very good accuracy.

Today physicists describe natural phenomena with mathematical models belonging to one of four compatible major classes of theories:

- Classical or Newtonian theory: this class contains mathematical models which describe the world at our scale, as we see with our own eyes and interact with our senses; these were the only known mathematical models until the beginning of the XXth century;

- Quantum theory: the mathematical models in this class were discovered in the first half of the XXth century and they describe physics at the atomic scale; today they have every day applications since for example our mobile phones and indeed any digital equipment[1] would not work without our understanding of Quantum theory and its predictions;

- Field theory: the mathematical models in this class were discovered in the second half of the XXth century and are still studied in the universities and research centres; they describe the physics of subatomic particles like the Higgs' particle recently discovered at CERN (Geneva CH);

- String theory and similar: the very recent mathematical models of this class probe the unknown of the infinitesimal small and exceedingly large (stars, black-holes and similar); as of today they are just theoretical elucubrations and for the moment we have no way of checking if they describe reality or not.

Notice that physicists like to classify physical phenomena using their (usually inner) energy scale. Instead we are used to classify objects by their dimensions. As a very hand-waving argument, we

1 One of the fundamental components of digital electronics and semi-conductor applications is the P-N Junction which is fully explained by Quantum theory.

can say that energy scale and physical dimension scale are inverse to each other:

- low energy scales correspond to every day (large) objects;
- medium energy scales correspond to atomic sized objects;
- high energy scales correspond to sub-atomic objects;
- very high energy scales correspond to sub-nuclear objects, eg. elementary particles, but also to stars and black-holes.

I finally come to the last concept in the title of this short essay: what does human intuition has to do with all of this?

We humans have evolved on Earth in a particular and well defined environment. We survived and became able to master our environment because during our evolution we adapted and became acquainted with it. Our brain has developed strong capabilities of analysing and understanding the physical phenomena which surround us. We would have not survived and evolved to this point otherwise.

Every man has a good intuition (some better than others) of the physical phenomena which surround us: for example we can easily imagine, guess and determine the trajectory of a thrown stone or arrow; we can drive a car keeping at the same time in consideration the trajectories of all other cars on the same road, and so on. For all of this we do not really need to do mathematical computations, our brain does them for us automatically. We are not even aware consciously that we are doing complex computations but we *feel* that we have an <u>intuition</u> of what is going to happen, and this intuition is usually correct.

Our intuition works fantastically well for most classical (eg. Newtonian) physics exactly because it is built into us. Most of the time we can trust it even though there are notable exceptions that we understand rationally, like for example the behaviour of frozen water which expands instead of contracting as it should do intuitively.

But what happens with physical phenomena at energy scales (or dimensions) different from that of our every day world?

Unfortunately our intuition miserably fails us.

This has been a very big issue in the physics community in the first half and middle part of the XXth century. The debates aroused by Niels Bohr and the Copenhagen school are probably the most notable examples, as it is the failure of Einstein to accept some of its consequences.

So in studying the very small (and very large) we must abandon our intuition and instead rely exclusively on our mathematical models and our rationality. Today physicists do the computations first and then study their results to rationally understand the physical phenomena. In these cases, *understanding* comes after math has done its job, not before (even if some scientists are very good in predicting the results of the mathematical computations, this is scientific expertise and not human every-day intuition). It is very important that these mathematical models give predictions which can be verified experimentally and that do not violate the predictions of other models for those phenomena that can be described by multiple models.

In this way we built a hierarchy of interdependent mathematical models which are verified experimentally (as far as possible) and do not contradict each other. We understand them rationally but they defy our every-day intuition.

Of course this makes life harder for scientists who should first learn to trust their math and abandon their intuition, and it makes a very good case for the impossibility of the dissemination of the results. Indeed if the audience does not have the mathematical tools to understand the workings of a mathematical model which describes a class of phenomena, how can you describe to it what is the most recent discovery without appealing to the human intuition?

Of course what happens is that in communicating to laymen the latest results in elementary particle physics, physicists do resort to description by analogy with classical (Newtonian) models and this can work fine as far as it is supposed to give a vague idea of what it is all about. But it carries a big risk: people can believe that they have an intuition on (or have understood) how a non-classical phenomenon works using a classical (Newtonian) description by analogy and try to use this intuition to extend this analogy to other non-classical phenomena. The result of this exercise is always wrong since one is trying to apply a theory outside is domain of application. This in turn can lead to very heated discussions and debates even within the scientific community, or to ill-fated efforts to reconstruct Quantum or Field theory by means of Classical mechanics.

This leaves us with a rather big problem: new discoveries, new concepts and the latest developments of science are a common treasure of the whole humanity so it is most important that the scientific community disseminates this knowledge to everyone. Unfortunately the only way we have of doing this is to resort to description by analogy and by relying on human intuition which in turn can easily lead to misunderstanding and misconceptions.